Indian Statistical Institute, Bangalore
B. Math (III)
Second Semester 2014-2015
Semestral Examination : Statistics (V)
Sample Surveys and Design of Experiments.
Date: 08-05-2015 Maximum Score 50 Duration: 3 Hours

1. Consider the following estimator of the *population mean* $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$

$$\widehat{\overline{Y}} = \begin{cases} \overline{y} + c & \text{if the sample contains unit 1} \\ \overline{y} & \text{otherwise,} \end{cases}$$

where $\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ is the sample mean and $c \geq 0$ is a constant. The value y_1 is known to be an unusually low y-value, this having come to light after the *simple random sampling without replacement (SRSWOR)* sample is drawn. Is $\widehat{\overline{Y}}$ unbiased for $\overline{Y}$? Show that $V(\overline{y}) \geq MSE(\widehat{\overline{Y}})$ if and only if $2\frac{N-n}{n(N-1)}\left\{\overline{Y} - y_1\right\} \geq c \geq 0$.

$[4 + 8 = 12]$

2. Explain the set up for *completely randomized design (CRD)* having a treatments with n replicates each. Derive the F-test for testing whether the treatments differ significantly.

$[14]$

3. Consider *balanced incomplete block design (BIBD) and randomized complete block design (RCBD),* both to study the effects of same set of a treatments. The two designs have the same total number of observations N and same error variance σ^2. Compare the two designs in terms of $Var(\widehat{\tau}_i - \widehat{\tau}_{i'})$, $i \neq i'$.

$[10]$

4. When and how would you use *Latin Square Design (LSD)?* Obtain *Least Squares Estimators (LSE)* of the model parameters and set up and carry out the desired statistical test.

$[3 + 7 = 10]$

5. Explain *2 factor complete factorial design* and the model used therein. Set up and carry out a test for interaction effect.

$[12]$